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Fluid discharge resulting from puncture of spherical process vessels

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Abstract

Risk analysis associated with incidents of puncture or rupture of process vessels generally requires estimation of actual or average fluid discharge rates resulting from such an incident. Most formulas developed to date for fluid discharge rates from vessels generally assume that the flow opening is located at the bottom of the vessel; this is undoubtedly due to the previously predominant interest in computing time requirements for gravity drainage of process or storage vessels. An accidental puncture, however, such as resulting from a moving vehicle, can occur at almost any elevation. Hence, from a risk analysis point of view, it would be useful to have formulas which would estimate fluid discharge amounts and rates from a flow opening at any arbitrary elevation. In this article, the differential and algebraic equations governing liquid discharge from an opening at any point on the surface of a spherical vessel are solved. This solution is then generalized in terms of dimensionless efflux times, liquid volumes and average release rates as functions of dimensionless elevations.

1. Introduction

The general subject of fluid flow rates from vessels, either storage or process, has posed numerous problems of engineering interest for many years now — to both practicing engineers concerned with production, inventory, safety and other aspects as well as to engineering professors as a source of practical mathematical material. Thus, papers [1–3] dealing with liquid efflux rates from some of the simpler and more common vessel shapes (e.g., vertical circular cylinders) date back to as early as 1949. These early efforts considered drainage through either a hole (e.g., orifice) or through a piping system, and for the most part addressed gravity flow only, ignoring any pressure head in the vessel. An exception here is the early work of Burgreen [3], as well as the much more recent work of Woodward and Mudan [4].

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Significant interest developed in this subject of tank drainage — perhaps at least partially from safety considerations — during the 1980s. Numerous articles appeared during this decade, many of them treating vessel shapes of more complex geometry, wherein the cross-sectional area formed by the liquid surface varies with the elevation of the latter. In one of the earlier such articles [5], formulas were summarized to compute the time requirements to empty vessels of four different shapes: vertical cylinder, cone, horizontal cylinder (with flat ends), and sphere. These expressions were all developed for drainage through a hole or orifice located at the bottom of the vessel. Later articles gave similar formulas for draining elliptical vessel heads often present at the bottom of vertical cylinders [6], elliptical saturator troughs (horizontal elliptical cylinders with flat ends) [7] such as employed in the textile finishing industry, and horizontal cylinders with elliptical dished heads or ends [8]. Some of these formulas are beginning to find their way into recent textbooks [9] on process safety.

Also in recent years the more complicated problem of vessel drainage through a piping system, originating at the bottom of the vessel, has been studied in considerable detail. Thus, one of the earlier articles [10] on this subject pertains to the simplest case of draining a vertical cylindrical tank (and hence of constant cross-sectional area) through such a piping system. Later works gave analogous formulas for draining spherical vessels [11], conical tanks [12] and horizontal cylinders [13], all through a piping system. Analytical solutions were obtained in all of these cases, although the mathematical expressions in the latter case include elliptic functions. A summary [14] of formulas to compute efflux times for drainage, either through a bottom drain hole or through a piping system originating at the vessel bottom, for about a dozen different vessel shapes was recently prepared.

As indicated above, however, all of these cited works have been concerned with fluid discharge rates from the bottom of a vessel, such as in a drainage situation. Some years ago there was an article [15], in which an approximate method for estimating fluid level changes in vertical cylindrical tanks with a multiplicity of outlets (or leaks), of various sizes and at different elevations, was presented. Because of increasing concerns about safety and loss prevention, there exists today a need for accurate formulas to compute fluid discharge and vessel emptying rates for an opening of a given size and at an arbitrary elevation. Such a need may arise, for example, in analyzing an accident scenario resulting from a moving vehicle, e.g., a forklift truck, being driven into the side of a vessel and creating an aperture for fluid discharge at some elevation. Thus, in this article the material and mechanical energy balance equations describing liquid efflux from a spherical tank under the influence of gravity are derived and solved. The solution is then generalized in terms of dimensionless variables, i.e., normalized efflux times, liquid volumes and average release rates as functions of dimensionless elevations.

2. Balance equations

The dynamic material balance for the liquid in the tank in this relatively simple case merely reduces to the rate of accumulation being equal to the negative of the output rate:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -q \tag{1}$$

or, more specifically:

$$A\frac{\mathrm{d}h}{\mathrm{d}t} = -A_0 v_2 \tag{2}$$

For the simpler geometric vessel configurations, e.g., vertical cylinders, the cross-sectional area (A) of the liquid surface in the vessel is a constant quantity and not a function of the variable liquid level (h). This results in a very tractable, non-linear differential equation.

The orifice equation is generally used to represent fluid discharge rates through openings in vessels, irrespective of their size, shape or location. Thus, the effluent liquid velocity (v_2) is represented by the following equation:

$$v_2 = C_0\sqrt{2g(h-h_0)} \tag{3}$$

where C_0 is known as the orifice discharge coefficient; it generally is a function of the fluid velocity (as incorporated in a Reynolds number) and the downstream (orifice)/upstream diameter ratio, although a constant value (between 0.60 and 1.0) is typically assumed for a given application. As indicated in Fig. 1, h_0 is the vertical elevation of the hole above the bottom of the sphere, and h is the variable elevation of the liquid level in the vessel. Equation (3), which derives from the Bernoulli equation, essentially equates the potential energy of the liquid head in the tank, represented by $(h-h_0)$, with the kinetic energy of the outflowing liquid, with any friction losses accounted for by C_0.

3. Mathematical solution

Insertion of eq. (3) into eq. (2) and rearrangement then leads to the following general expression to be integrated:

$$t = \frac{1}{C_0 A_0 \sqrt{2g}} \int_{h_0}^{h_1} \frac{A}{\sqrt{h-h_0}}\,\mathrm{d}h \tag{4}$$

in order to determine the time (t) required for the liquid level to fall from its initial elevation of h_1 to the elevation of the discharge hole h_0. The area (A) formed by the liquid level in this case of a spherical vessel is given by $\pi C^2/4$, where C is the length of the chord formed by the liquid level. It can simply be shown with the aid of the Pythagorean Theorem that this latter quantity is

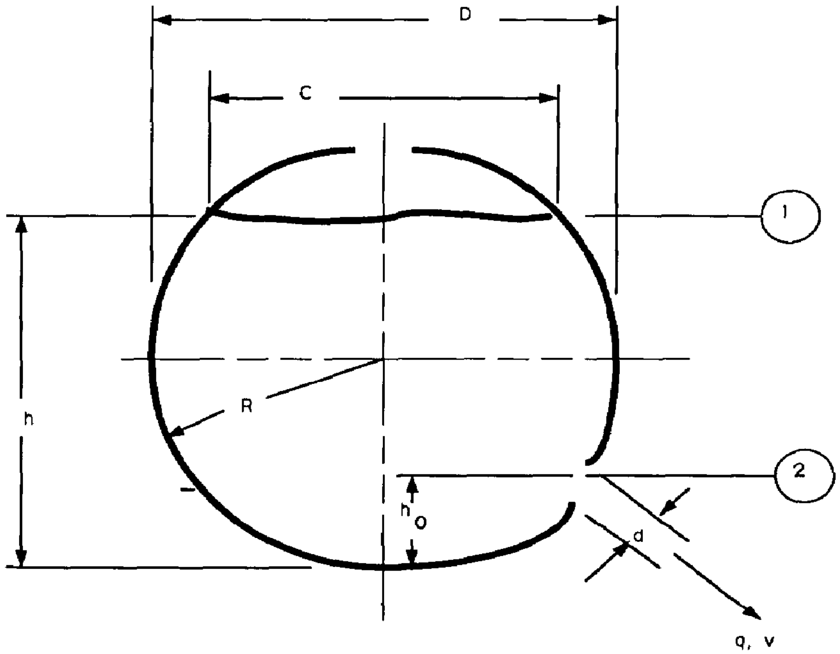

Fig. 1. Sketch of a spherical vessel with a puncture hole in its side and resulting liquid drainage.

given by $2(hD-h^2)^{1/2}$. Equation (4) then becomes:

$$t=\frac{\pi}{C_0A_0\sqrt{2g}}\int_{h_0}^{h_1}\frac{hD-h^2}{\sqrt{h-h_0}}\,\mathrm{d}h \tag{5}$$

as the specific expression to be integrated.

The integration of eq. (5) can be found in most tables of integrals. In this particular case the lower limit of h_0 (corresponding to the elevation of the hole and thus the end of the discharge process) vanishes, and there results:

$$t=\frac{2\pi}{15C_0A_0\sqrt{2g}}\,[5Dh_1+10Dh_0-3h_1^2-4h_1h_0-8h_0^2]\sqrt{h_1-h_0} \tag{6}$$

When $h_0=0$, as in conventional drainage from a hole or orifice in the bottom of a spherical tank, eq. (6) becomes:

$$t=\frac{\sqrt{2}\,\pi}{3C_0A_0\sqrt{g}}\left[\left(D-\frac{3h_1}{5}\right)h_1^{3/2}\right] \tag{7}$$

which is the same expression as presented by Foster [5] in his summary article.

4. Generalization of solution

In order to impart greater generality and usefulness to eq. (6) above, let us convert that equation to dimensionless form by the introduction of several dimensionless variables. Specifically, let us define two dimensionless elevations, one corresponding to the initial liquid level in the spherical vessel:

$$x_1 \equiv h_1/D \tag{8}$$

and the other representing the elevation of the discharge hole:

$$x_2 \equiv h_0/D \tag{9}$$

It is necessary to define two more dimensionless quantities. The first of these is the dimensionless size of the flow opening, represented as follows:

$$x_3 \equiv d_o/D \tag{10}$$

and the other is a dimensionless time, given by the following expression:

$$Y \equiv \sqrt{\frac{2g}{D}}\, t \tag{11}$$

After insertion of the dimensionless quantities from eqs. (8)–(11) into eq. (6), the latter equation can be written in the following form:

$$Y = \frac{8}{15C_0x_3^2}\,[5x_1 + 10x_2 - 3x_1^2 - 4x_1x_2 - 8x_2^2]\sqrt{x_1 - x_2} \tag{12}$$

There are four independent parameters appearing in eq. (12) — x_1, x_2, x_3 and C_0, in addition to the dimensionless dependent variable of the time (Y) required for vessel drainage. A commonly assumed value for the orifice discharge coefficient (C_0) is 0.60. Thus, for a given dimensionless opening size (x_3), one can construct a graph which is based upon eq. (12) and which depicts the behavior of the time required for complete drainage of the vessel from an initial dimensionless liquid level of x_1 down to the dimensionless elevation of the opening. Figure 2 represents such a graph for fixed values of $C_0 = 0.60$ and $x_3 = 0.005$; the latter value might correspond, for example, to a hole 5 cm in diameter in the side of a spherical vessel with a diameter of 10 m. This graph shows how the time required for tank drainage increases with the amount of liquid contained in the tank (as measured by x_1), as well as with increased elevation of the drainage hole (x_2); this latter effect undoubtedly results from the reduced hydrostatic head as x_2 increases. The crossover in the curves for the higher values of x_2 (> 0.30) is interesting.

The actual amount of fluid released to the environment is generally of considerable interest in any accident scenario. Again because of the more

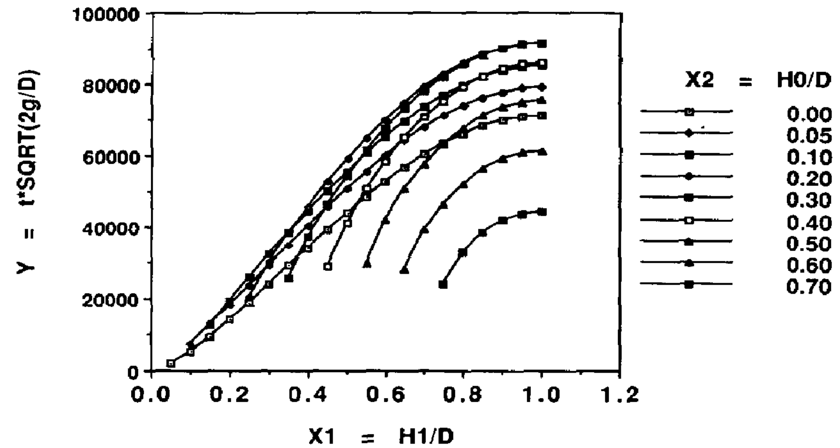

Fig. 2. Dimensionless time (Y) for complete drainage of a spherical vessel as a function of dimensionless initial liquid elevation (x_1) and elevation (x_2) of the drainage hole. $x_3 = 0.005$; $C_0 = 0.60$.

complex geometry of spheres, the calculation of such amounts is somewhat more complicated than for vessels of constant cross-sectional areas, such as vertical circular cylinders. None the less, formulas do exist for calculating the volume of spherical segments. Thus, the volume (V) of a segment h units high of a sphere with a diameter of D is:

$$V = \frac{\pi h^2}{6}[3D - 2h] \tag{13}$$

Recognizing that the complete volume (V_s) of a sphere of diameter D is given by $V_s = \pi D^3/6$, the segmental volume from eq. (13) can be normalized to the complete sphere volume to yield a dimensionless segmental volume:

$$\frac{V}{V_s} = x^2\,[3 - 2x] \tag{14}$$

where $x = h/D$ and corresponds to any liquid elevation.

The total amount of fluid released in a given incident will then be the difference in volumes corresponding to the initial spherical segment and that corresponding to the final spherical segment at the elevation of the discharge hole. Denoting this fluid volume, dimensionless and normalized to V_s, by U, we have:

$$U = x_1^2\,[3 - 2x_1] - x_2^2[3 - 2x_2] \tag{15}$$

Figure 3 then shows how this dimensionless volume U varies with the two dimensionless elevations — x_1 and x_2. Not surprisingly, U increases monotonically with increasing x_1 and decreases monotonically with increasing x_2,

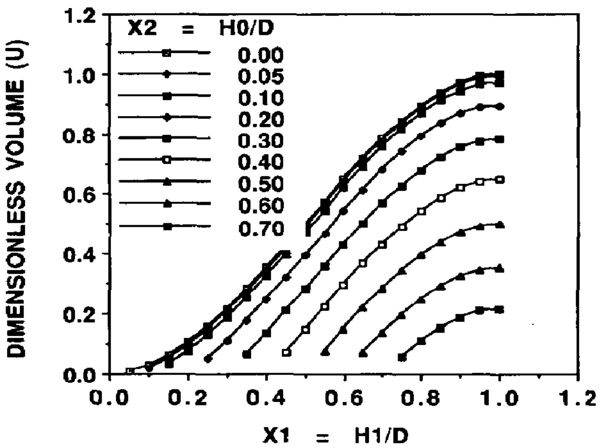

Fig. 3. Dimensionless volume (U) of liquid stored in a spherical vessel between some dimensionless initial liquid elevation (x_1) and elevation (x_2) of a drainage hole.

and with the greatest slopes occurring near the middle bulge ($x_1 = 0.5$) of the spherical vessel.

The curves of Fig. 3 are based solely upon the static geometric calculations of eqs. (13)–(15), and are thus valid for any spherical vessel. That is, the volume of a spherical segment with two horizontal bases at arbitrary elevations can be readily computed from these equations. Flow dynamics add another dimension to the problem. Consider the related task of determining the time required for the spherical vessel to drain from some initial liquid level elevation of h_1 down to some intermediate level of h_i through an opening of diameter d located at a lower elevation of h_2. In this case the volume drained will still be given by eqs. (13)–(15), evaluated for x_1 and $x_i = h_i/D$; but determination of the time requirement here necessitates two evaluations of drainage times from eq. (6):

$$t(h_1 \to h_i) = t(h_1 \to h_0) - t(h_i \to h_0) \tag{16}$$

Alternately, the above calculations can be done in dimensionless form via two applications of eq. (12).

Some representative volumetric rate of fluid discharge from the vessel is often needed in the analysis of puncture accident scenarios. According to Woodward and Mudan [4], a reasonable such rate to use is roughly midway between the initial and average discharge rates. The initial rate is readily obtained from eqs. (1)–(3), with h set equal to h_1. Evaluation of the average rate, however, requires integration of the differential balance equations, as performed in this article. Specifically, this average rate is given by the total volume of liquid contained in the spherical segment (between h_1 and h_0) of interest divided by the time required to drain this liquid through the puncture hole. In dimensionless form, this average rate can be expressed as follows:

$$W = U/Y \tag{17}$$

A plot of W versus x_1 for various values of x_2 ($<x_1$) is shown as Fig. 4; this figure, like Fig. 2, is for values of $C_0=0.60$ and $x_3=0.005$. Figure 4 is somewhat similar to Fig. 3 for the dimensionless volume U, in that the average dimensionless discharge rate (W) generally increases with increasing x_1 and decreases with increasing x_2.

5. Example calculations

Let us illustrate the calculation procedure outlined above with the following example. We consider a spherical storage vessel, vented to the atmosphere, with a diameter of 10 m and initially filled with a corrosive solution to a depth of 7 m. A forklift truck is accidentally driven into this vessel, creating a puncture hole of about 5 cm in diameter 2 m above the vessel bottom. Assuming uncontrolled release of fluid from this vessel down to the puncture location, it is desired to determine how much fluid is released over what period of time, and hence the average discharge rate. A constant orifice discharge coefficient of $C_0=0.60$ is to be used for these calculations.

The dimensionless elevations are readily computed as $x_1=0.70$ and $x_2=0.20$. Similarly, the dimensionless orifice size (x_3) is found to be 0.005. For this particular configuration, the factor of $8/(15C_0x_3^2)$ is then computed as 35,556. Inserting these various values into eq. (12), we find a dimensionless drainage time of $Y=79{,}200$. With $g=9.807$ m/s^2 and $D=10$ m, this dimensionless drainage time translates to a value of 56,550 s. Similarly, from eq. (15) the total dimensionless volume (U) of fluid drained over this time interval is 0.680 (of the complete volume of a sphere with a diameter of 10 m). Thus, this dimensionless volume corresponds to a fluid volume of 356 m^3. Lastly, the dimensionless

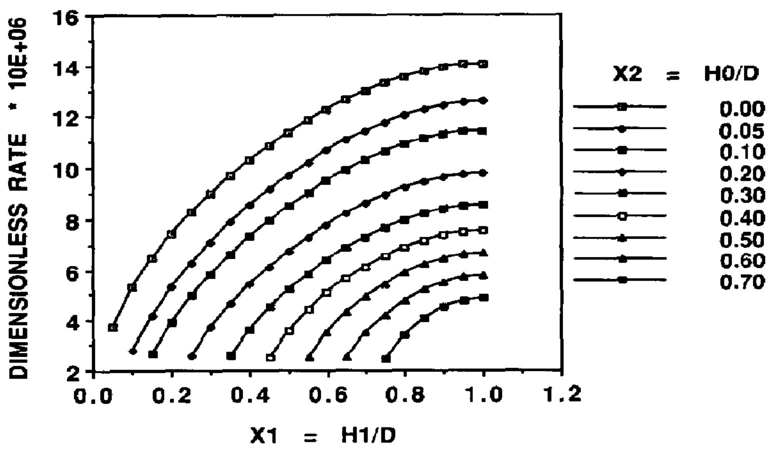

Fig. 4. Dimensionless average rate ($W=U/Y$) of discharge from a spherical vessel as a function of dimensionless initial liquid elevation (x_1) and elevation (x_2) of the drainage hole. $x_3=0.005$; $C_0=0.60$.

average draining rate from eq. (17) is 8.59×10^{-6}, while the actual average draining rate is $(356\,m^3/56{,}550\,s) = 0.00629\,m^3/s = 22.7\,m^3/h$. This latter quantity compares with an initial discharge rate, as evaluated from eqs. (1)–(3) with $h = 7.0\,m$, of $42.0\,m^3/h$. The middle value between these two discharge rates is then about $32.3\,m^3/h$.

6. Discussion

While the mathematical analysis and derivation presented above are rigorous, several of the assumptions made in the early formulation of this problem of drainage from a ruptured sphere may be questioned. Thus, the selection of a given value for the orifice discharge coefficient (C_0) and the assumption of a constant value for this coefficient merit some attention. As can be found in any standard text on the unit operations of chemical engineering [16], this discharge coefficient varies with the Reynolds number for the fluid stream flowing through the opening as well as with the orifice diameter/upstream diameter ratio. It is well known [16] that 0.61 is a reasonably constant value for the discharge coefficient of a sharp-edged orifice when the Reynolds number through the orifice exceeds 30,000, irrespective of the ratio of the diameters. If the orifice is not sharp-edged but rounded at the upstream face (highly unlikely in the event of a puncture hole), the discharge coefficient has a value varying from 0.70 to 0.88. Given that any aperture accidentally created in the side of a process vessel is likely to be highly jagged in nature, a value of 0.61 for C_0 would appear to be a high-side estimate. The value of 30,000 is not a terribly large value for the Reynolds number, and hence the assumption of a constant value for C_0 should be valid for the vast majority of all fluids with viscosities close to water; certainly, exceptions here might include ethylene glycol and kerosene. Also, there might be deviations in the value of C_0 as the drainage of fluid neared its termination, but such deviations would impact upon only a small fraction of the total amount of fluid discharged.

One method of addressing the problem of a variable value of C_0 would be to break up the problem into several segments for values of the liquid elevation (h), and to employ different values of the discharge coefficient for these various segments. This approach would then require successive application of eqs. (6) and (16) to these segments, for h_1, h_2, h_3 . . . Admittedly, this procedure could shortly become very cumbersome, and one might be better advised to employ some computerized numerical integration scheme to solve the original differential equation.

Secondly, estimation of the actual flow area (A_0) associated with a puncture hole could prove somewhat difficult. One method for estimating such a puncture area might derive from mechanical calculations involving the vessel shell thickness and mass and velocity of the moving object, among other variables. If puncture by a forklift truck is indeed the concern, some estimate of the opening might be made from the cross-sectional area of a truck prong. In any event, there is no requirement in this analysis that the puncture hole indeed be circular, as eq. (10) might suggest. Given an estimate of the flow area (A_0), an equivalent diameter (d_0) for the discharge opening can always be readily computed if dimensionless charts are to be used.

Lastly, there is the question of an internally pressurized fluid, which this article does not specifically address. In the simpler case wherein this internal pressure is constant, such as resulting from a tank pressure controller or the vapor phase over a volatile liquid, this constant tank pressure head may simply be added to the hydrostatic liquid head, $2g(h-h_0)$, in eq. (3); Woodward and Mudan [4] employed this procedure to develop equations for calculating liquid and gas discharge rates through holes in the bottom of process vessels of different shapes. If this tank pressure head decreases as the vessel drains (as in the case of an unvented vessel), however, the governing differential equation in all but the simplest geometric configurations (vertical cylindrical) becomes quite cumbersome.

7. Conclusions

The engineering equations describing fluid discharge from a hole of arbitrary size and at any location on a spherical vessel can be readily solved; the vessel can be of any diameter and initially filled to an arbitrary liquid level.

The mathematical solution obtained to these equations can be generalized in terms of dimensionless variables. This solution can be useful in risk analyses of scenarios associated with accidental puncture of spherical storage or process vessels, such as might result from impaction by a moving vehicle, e.g., a forklift truck. Specifically, this solution allows computation of the amount of fluid discharged to the environment, duration of such fluid discharge and average discharge rate over this time period.

Nomenclature

A cross-sectional area of the liquid surface in the vessel at any time, m^2
A_0 cross-sectional area of the hole for liquid flow out of the vessel, m^2
C length of chord formed by liquid level in vessel, m
C_0 orifice discharge coefficient
D diameter of spherical vessel, m
d diameter of the hole for liquid flow out of the vessel, m
g acceleration due to gravity, m/s^2
h elevation of liquid level in vessel at any time, m
h_i intermediate elevation of liquid level in vessel, m
h_0 elevation of hole above the bottom of the spherical vessel, m
h_1 elevation of initial ($t=0$) liquid level in vessel, m
q liquid volumetric flow rate out of the vessel, m^3/s

R radius of spherical vessel, m
t time, s
U dimensionless volume of liquid initially contained in vessel
V liquid volume in the vessel, m^3
v_2 linear velocity of liquid out of the vessel, m/s
W dimensionless average rate of vessel drainage ($=U/Y$)
x dimensionless elevation of liquid level ($=h/D$)
x_i dimensionless elevation of intermediate liquid level ($=h_i/D$)
x_1 diensionless elevation of initial liquid level ($=h_1/D$)
x_2 dimensionless elevation of hole in vessel ($=h_0/D$)
x_3 dimensionless inside diameter of hole in vessel ($=d/D$)
Y dimensionless time for complete drainage of vessel ($=t(2g/D)^{1/2}$)
π number pi (3.14159 . . . .)

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